
RandLinearAlgebra

If the docker is not working

You can find all the python notebooks here:

(`https://github.com/nathanielpritchard/RandLinearAlgebra_examples`)

Contributors

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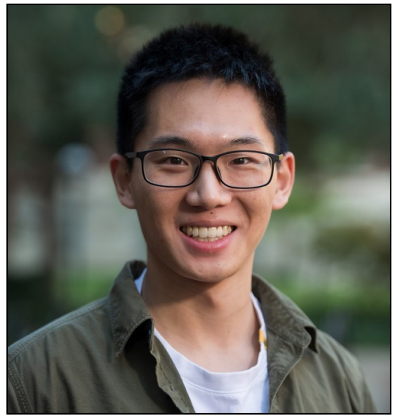
Adrian Maldonado



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This project is supported by NSF Awards 2309445 and 2309446.

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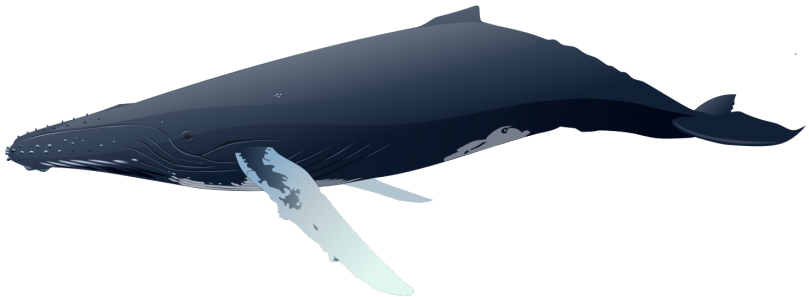

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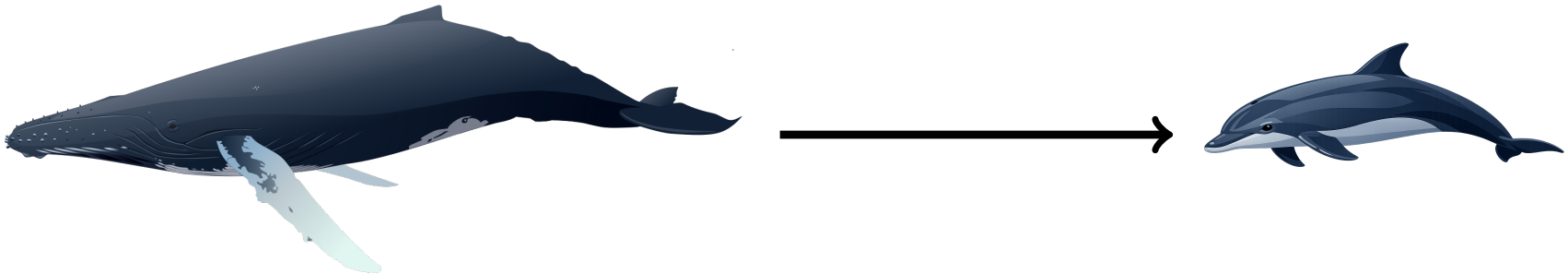

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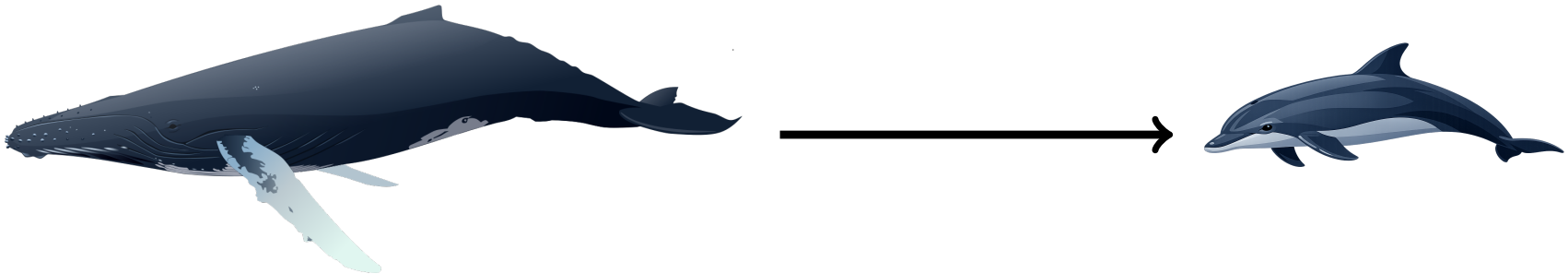

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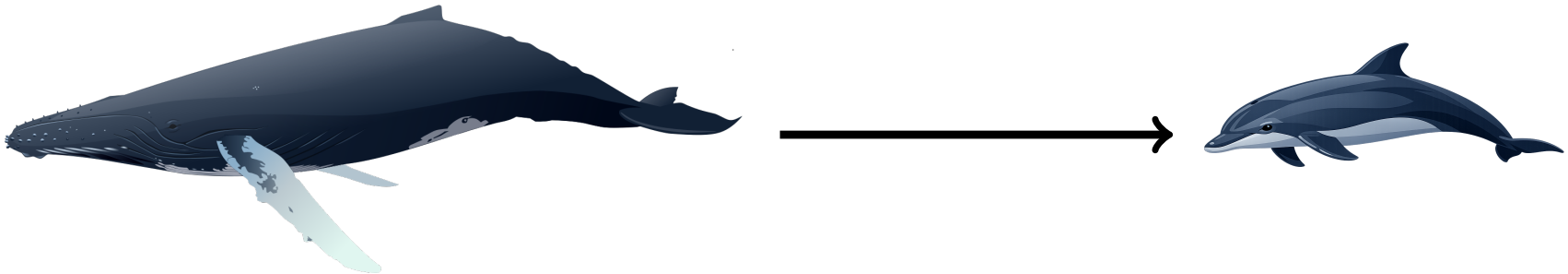

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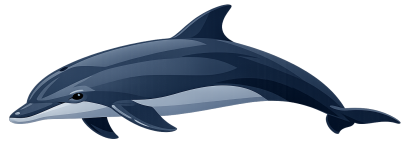
SVD()

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Theory often tells us how one set of choices performs on the worst possible problem. But,

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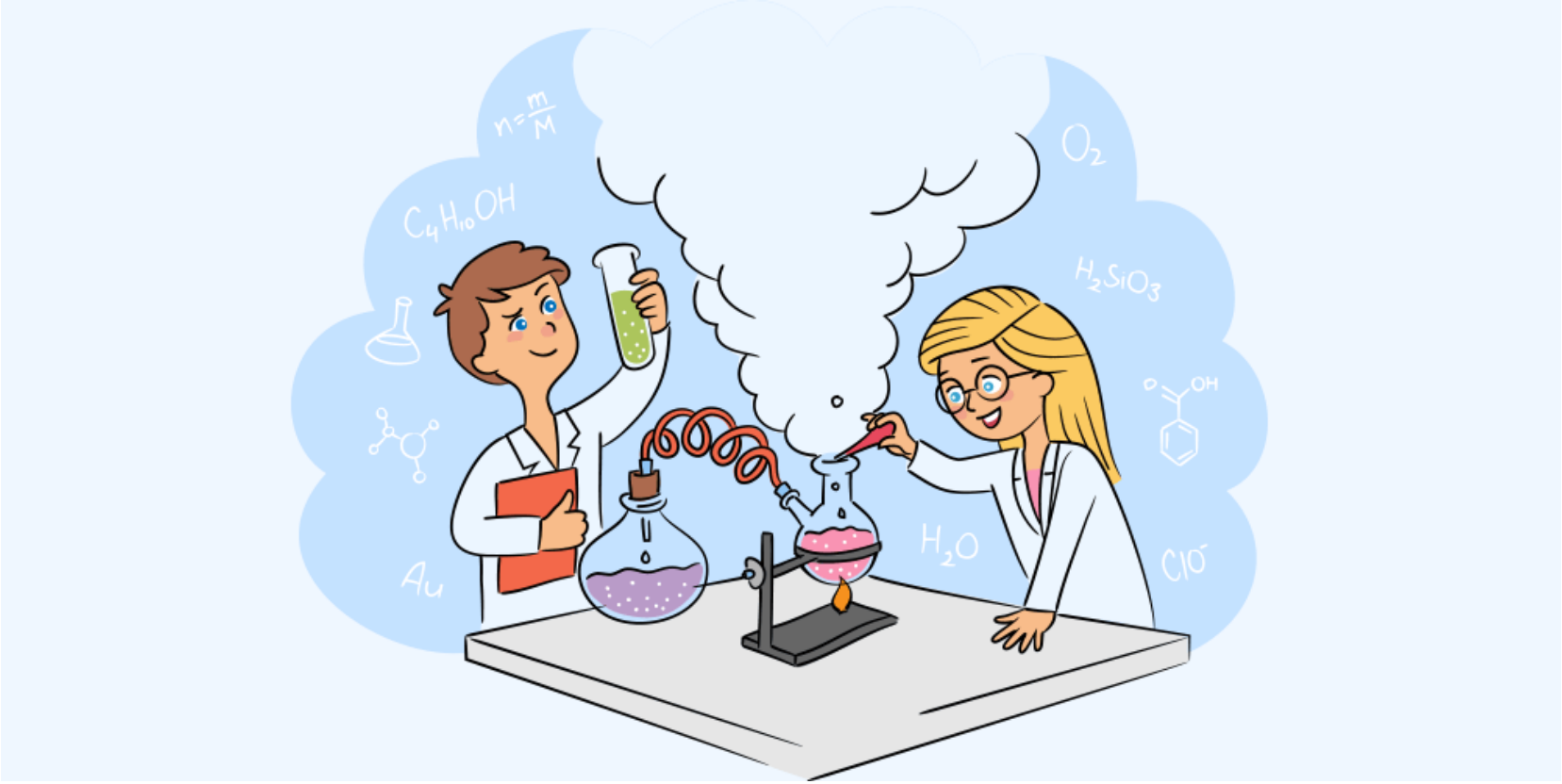
Then how should I implement these algorithms?

You just have to try...



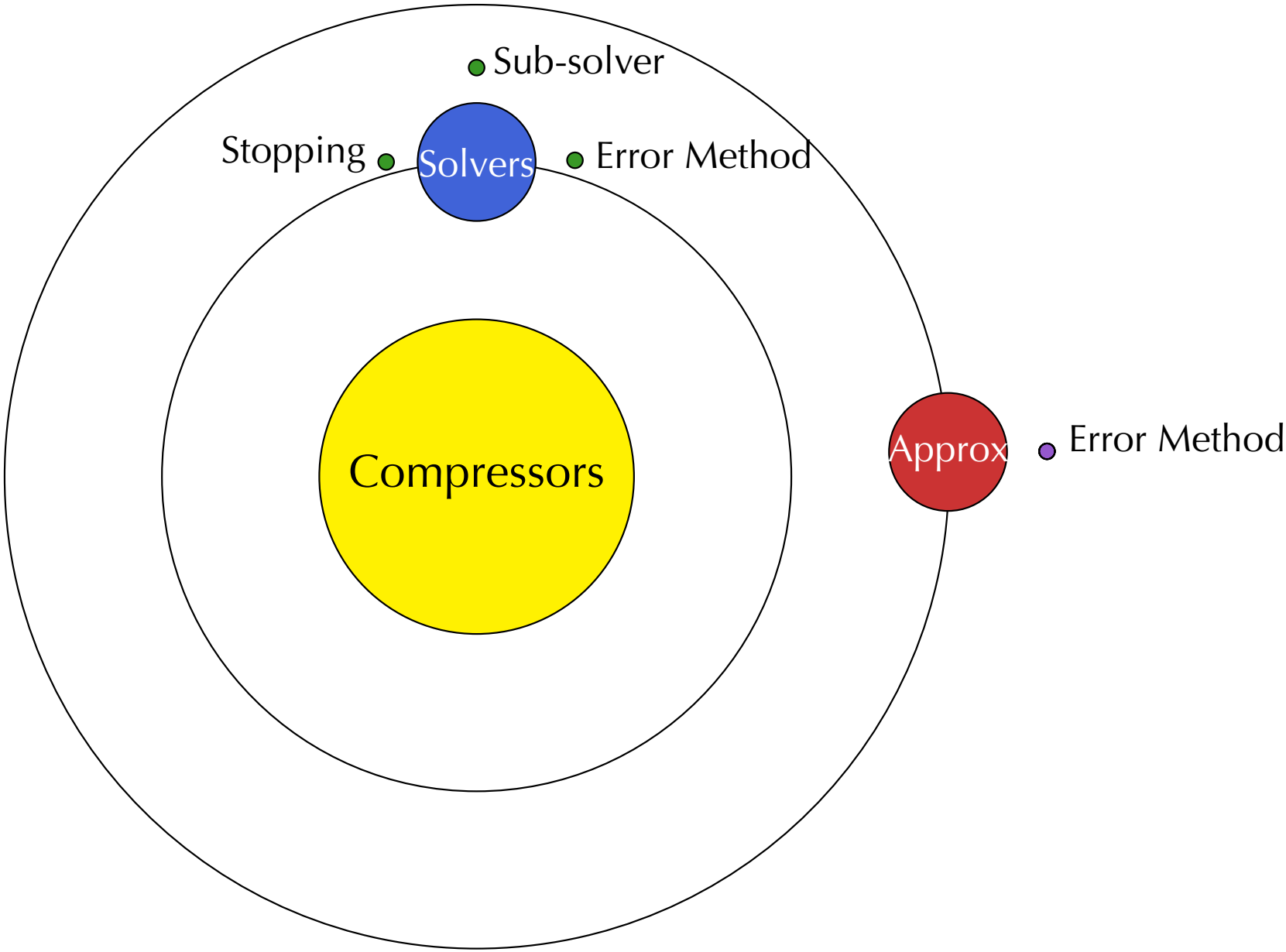
RandLinearAlgebra.jl: A library for rapid prototyping

- RandLinearAlgebra.jl is a **modular** Julia library
- Allows for **easy, in-depth** customization of RandNLA routines
- Can be plugged into established Julia codebases
- Julia also makes it easy to test on GPUs and with different levels of precision

Goals for this talk

1. Understand the structure of `RandLinearAlgebra.jl`
2. Understand how to use the library
   - **Attention** (How to compress a matrix)
   - **Preconditioning** (How to Solve a linear system)
   - **Operator Inference** (How to Approximate a matrix)

The Solar System of RandLinearAlgebra.jl Methods



Key Structures in `RandLinearAlgebra.jl`

Ingredient Structure:

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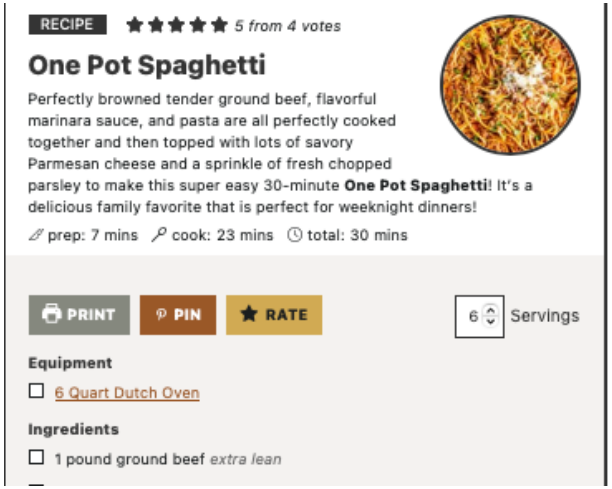
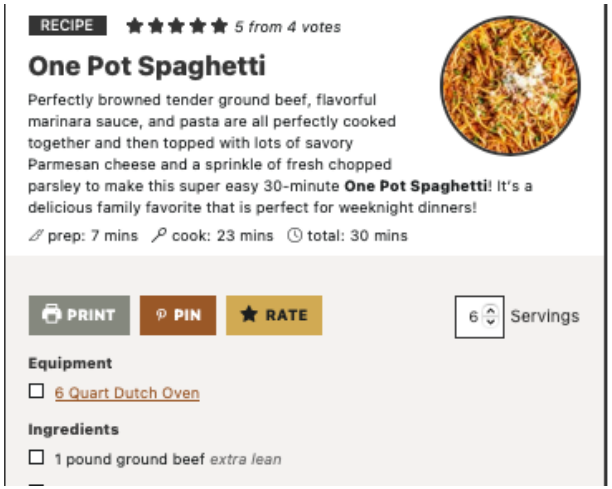
Recipe Structure:

- Denoted with the [Ingredient]`Recipe`
- Contains all the information necessary for carrying out the technique



How to get a recipe



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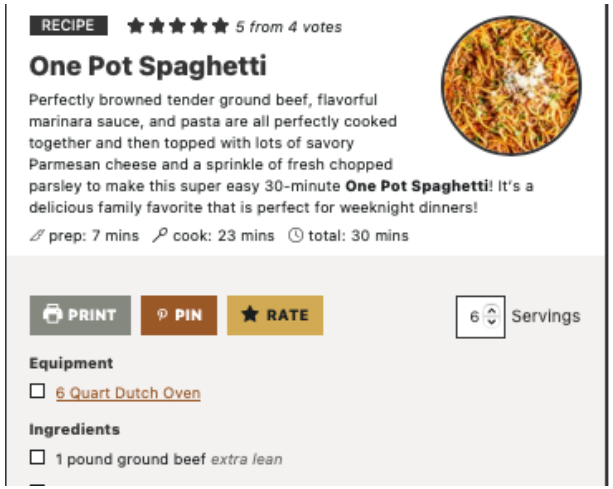
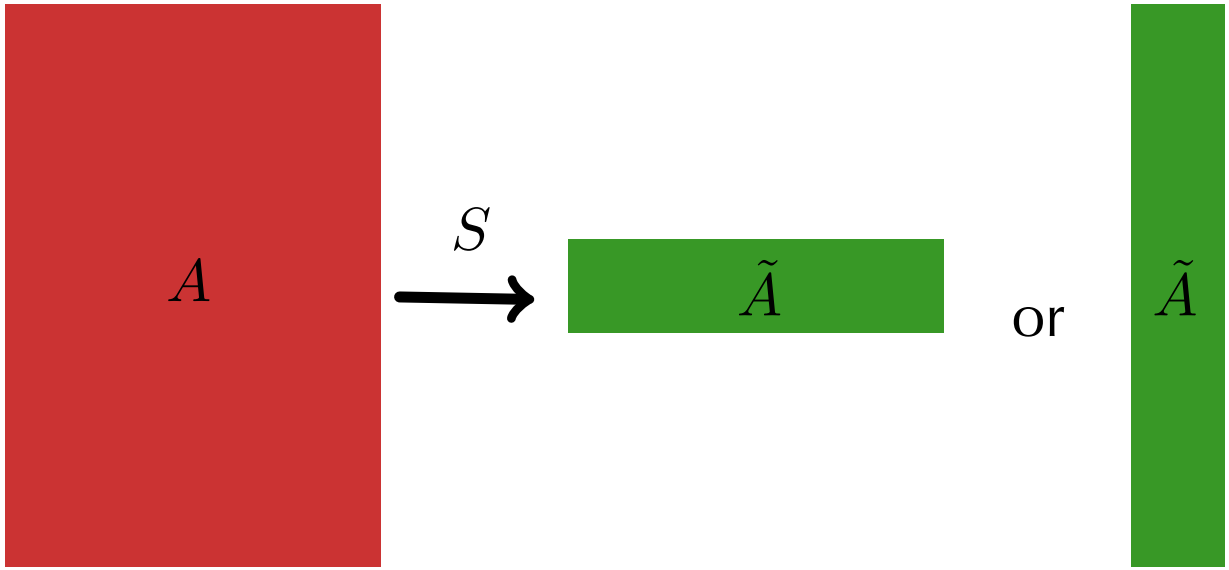


In `RandLinearAlgebra.jl` you just need to call the `complete_`[dataType]

Compressors



Left Compression
$\tilde{A} = SA \in \mathbb{R}^{p \times n}$

Right Compression
$\tilde{A} = AS \in \mathbb{R}^{m \times p}$

Types of Compressors

Sampling:

- S is a random subset of identity
- Random Subset chosen by sampling from a particular distribution
  - uniform distribution
  - leverage scores
  - determinantal point process
  - column/row norms

Sketching:

- Random linear combinations of rows/columns of the matrix
- Combination can arise from
  - Gaussian matrices
  - Randomized Trigormetric Transforms
  - Random Sparse matrices

Functions and Sub Structures

Functions:

- `complete_compressor`: turns your ingredients into a recipe
- `update_compressor!`: updates the randomness in your recipe

Sub Structures:

- `Cardinality`: Which side you primarily aim to apply the sketch
- `Distributions`: How you specify what you will sample indices from

Forming a Sketching Compressor

Decide:

- The method you want to use: **SparseSign**
- The size of compression dimension: **4**
- Whether we primarily want to apply from left or right: **Left**

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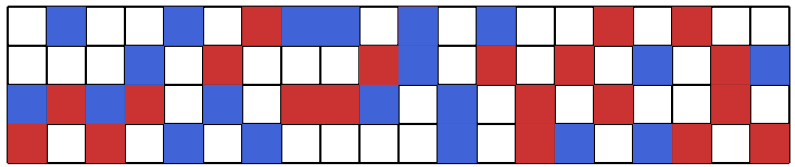

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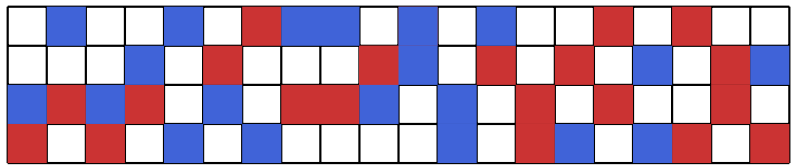

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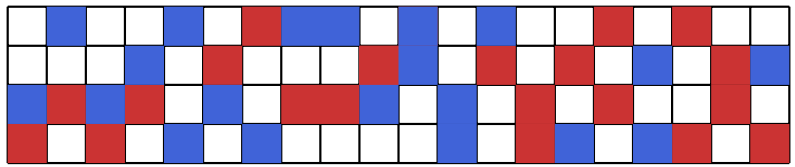

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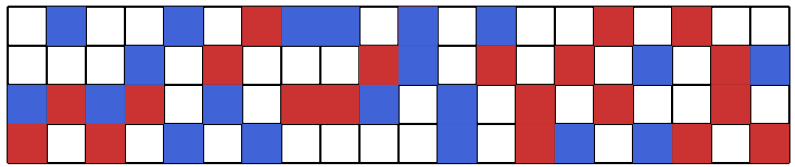

```
recipe = complete_compressor(
 SparseSign(
   compression_dim = 4,
   cardinality = Left()
 ),
 A
)
```

Forming a Sampling Compressor

- The method you want to use: **Strohmer Vershynin**
- The size of compression dimension: **4**
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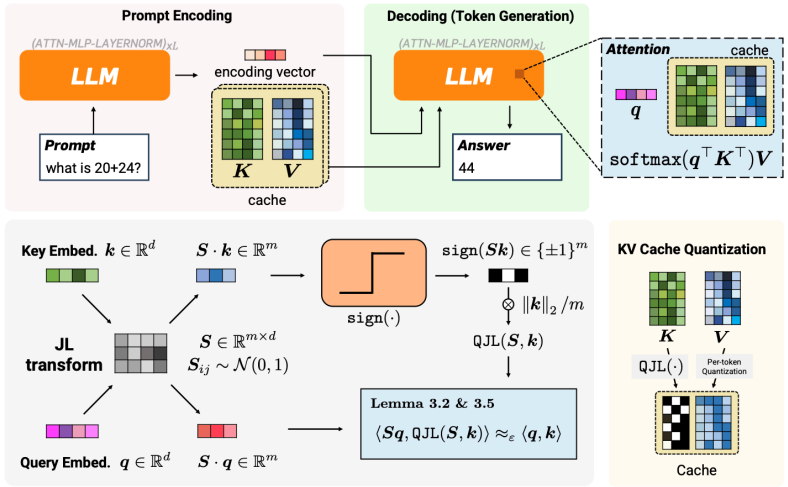
```
recipe = complete_compressor(
 Sampling(
   compression_dim = 4,
   cardinality = Left(),
   distribution = L2Norm()
 ),
 A
)
```

Example Usage: Attention



- The size of the Key Value matrices makes deploying LLMs really expensive
- Google proposed using random compression plus quantization to reduce the size of these Key-Value matrices Zandieh et al. [2024]
- In our first example we show how to use compressors RandLinearAlgebra to speed up the matrix-matrix multiplication

Solvers

Want to find an x^* such that

$$Ax^* = b \tag{1}$$

or

$$x^* = \text{argmin}_x \|Ax - b\|_2 \tag{2}$$

Types of Solvers:

- **Consistent Systems:**
  - Kaczmarz
  - Randomized Krylov Methods
  - Nystrom/CUR preconditioners

- **Least Squares Solvers:**
  - Coordinate Descent
  - Iterative Hessian Sketch
  - Blendenpik, LSRN
  - Sketch and Solve
  - FOSSILs

Functions and Sub Structures

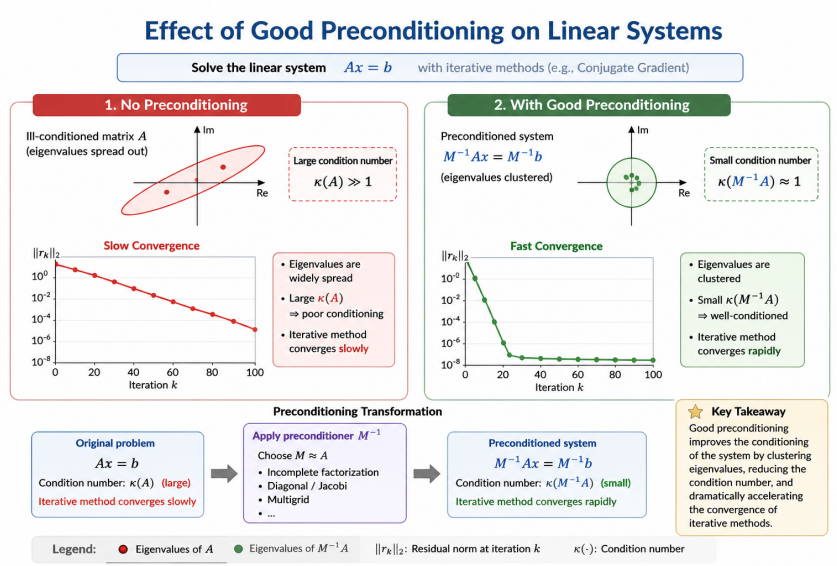
Functions:

- `complete_solver`: turns your ingredients into a recipe
- `rsolve(!)`: solves your linear system

Sub Structures:

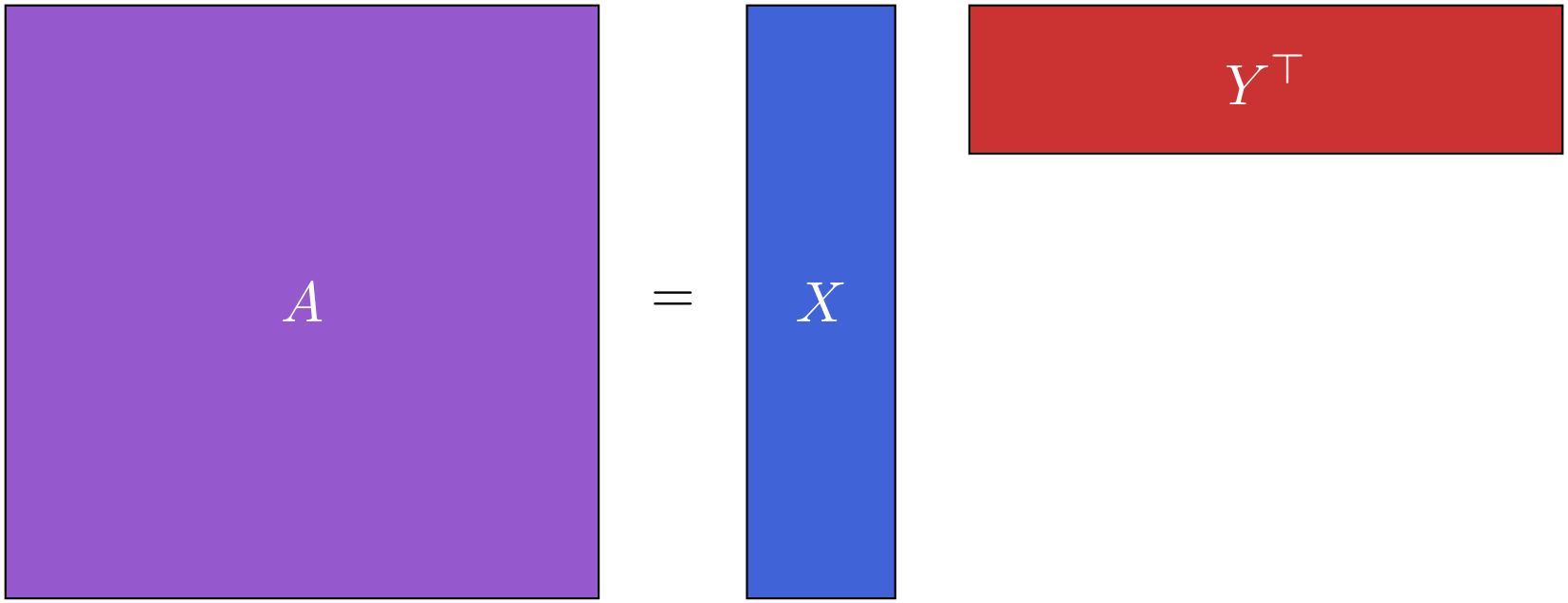
- `ErrorMethod`: Technique for measuring the accuracy of your solution
- `SubSolver`: How you will solve the compressed linear system (if necessary)
- `Logger`: How you log the progress of your solver and stop it when desired

Example Usage: Preconditioning CG



- We can use randomized solvers to precondition Krylov methods
- Now we show how we can incorporate the Kaczmarz solver to precondition CG

Approximators



Types of Approximators

Orthogonal Projection:

- Randomized RangeFinder (Q)
- RandSVD

Oblique Projection:

- Nystrom, Generalized Nystrom
- CUR, Interpolative Decomposition

Functions and Sub Structures

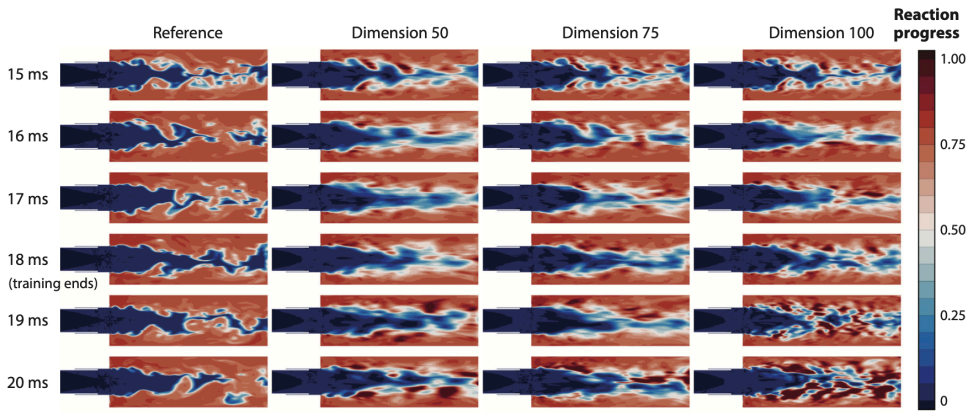
Functions:

- `complete_approximator`: turns your ingredients into a recipe
- `rapproximate(!)`: approximates your linear system

Sub Structures:

- `ErrorMethod`: Technique for measuring the accuracy of your approximation
- `Selector`: Technique for selecting indices in CUR/ID

Example Usage: Operator Inference



- Surrogate modeling approach that
  1. Projects a set of trajectories into a lower dimensional space
  2. Solves a least squares problem find the coefficients of a polynomial approximation of the trajectories Kramer et al. [2024]
- Consider Operator Inference applied to the one dimensional heat equations
- Will compare the subspace produced by `RandSVD` to that of the SVD

RandLinearAlgebra.jl makes prototyping easy

- The modular design makes it easy to test different variants of a RandNLA routine
- Julia makes it easy to test GPU performance and the use of different precisions
- Need relatively few lines of code to incorporate RandNLA routines into complex code

Check out the github!

Test out on Julia using:

```
]add RandLinearAlgebra.jl
```

or

```
using Pkg;
Pkg.add("RandLinearAlgebra")
```

Questions?

References I

Boris Kramer, Benjamin Peherstorfer, and Karen E. Willcox. Learning Nonlinear Reduced Models from Data with Operator Inference. *Annual Review of Fluid Mechanics*, 56(1):521–548, January 2024. ISSN 0066-4189, 1545-4479. doi: 10.1146/annurev-fluid-121021-025220. URL `https://www.annualreviews.org/doi/10.1146/annurev-fluid-121021-025220`.

Amir Zandieh, Majid Daliri, and Insu Han. QJL: 1-Bit Quantized JL Transform for KV Cache Quantization with Zero Overhead, July 2024.